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A unified nomenclature for project scheduling problems (RCPSP and RCMPSP)

Una nomenclatura unificada para problemas de programación de proyectos (RCPSP and RCMPSP)

Félix A. Villafáñez, David Poza, Adolfo López-Paredes, Javier Pajares

INSISOC – University of Valladolid. Paseo del Cauce 59, Valladolid, 47011. Spain

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Abstract: We present a unified formulation for the single and the multi-project scheduling problems (RCPSP and RCMPSP, respectively). Although this issue had been widely described in the literature, we found that, to our knowledge, the nomenclature used for the formulation of both problems has never been completely unified, which has traditionally hindered a comparison between approaches. For this reason, in this article we propose a unified nomenclature for the formulation of both problems

Keywords: Management, Project Scheduling, Multi-project Scheduling, RCPSP, RCMPSP

Resumen: . Presentamos una formulación unificada para el problema de programación con recursos limitados en entornos mono y multi-proyecto: Resource-Constrained Project Scheduling Problem (RCPSP) y Resource-Constrained Multi-Project Scheduling Problem (RCMPSP), respectivamente. Aunque este tema ha sido tratado ampliamente en la literatura, no nos consta que la nomenclatura empleada para la formulación de ambos problemas haya sido unificada de manera completa, lo que tradicionalmente ha dificultado la comparación entre distintas aproximaciones. Por esta razón, en este artículo proponemos una nomenclatura unificada para ambos problemas.

Palabras clave: Gestión de Proyectos, Programación de Proyectos, Programación multi-Proyecto, RCPSP, RCMPSP.

1. Introduction

Several simple but still well-known scheduling tools, such as Gantt Charts (Gantt 1913, 1919) or Milestone Diagrams, spread during the first half of the 20th century. However, these methods were only able to solve the most basic case of project scheduling: non-interruptible activities, activities with a single execution mode (i.e. the activities can only be executed with a concrete amount of resources) and a zero-lag finish-start precedence relation (i.e. an activity can only start right after her predecessor has finished).

Furthermore, these traditional methods did not take into account the difficulty stemmed from a limited availability of resources for the execution of the activities. If two or more activities require the same resource at the same time, the execution of some of the activities may be delayed until the resources they need are available. As a result, the scheduling generated by these classic methods turned out to be unfeasible in real-life projects (Araúzo et al. 2010; Villafañez et al. 2014).

This circumstance gave rise to a research field commonly referred to as resource-constrained project scheduling problem (RCPSP) Pritsker, Watters, & Wolfe (1969). The RCPSP has been intensively researched over the last few years. An extensive review of the RCPSP and a classification of the variations alternatives for its resolution can be found in Hartmann and Briskorn (2010). The RCPSP can be formally defined as an Integer Linear Programming model (ILP) or a Multiple Integer Linear Programing (MILP) in which there is an objective function (i.e. the project's feature to optimize) and a set of constraints such as the precedence relations of the activities and the limited availability of resources (Demeulemeester and Herroelen 2002b). However, it was demonstrated that the RCPSP was a non-deterministic polynomial-time hard (NP-hard) problem (Blazewicz et al. 1983), which means that it cannot be solved in an efficient and reasonable time due to its combinatory difficulty. Since then, the scientific community started to develop heuristic procedures (Zuloaga 2017), which permit obtaining a sufficiently good solution rather than the optimal one.

The resource-constrained multi-project scheduling problem (RCMPSP) is an extension of the RCPSP where some resources are shared among two or more projects run by the company (Fendley 1968; Davis

1969). For a complete and recent literature review of the alternative methods for the resolution of the RCMPSP please refer to Zuloaga (2017). Notice that because the RCMPSP is a generalization of the RCPSP, it is also NP-hard.

In this paper, we will focus on the formalization of these two scheduling problems. Although many articles in the literature are devoted to the formulation of these problems (Pritsker et al. 1969; Alvarez-Valdés Olaguíbel and Tamarit Goerlich 1993a; Kaplan 1996; Mingozzi et al. 1998; Klein 2000), as far as we know, the nomenclature used for the description of both problems has never been completely unified, which hinders the comparison between approaches. For this reason, in this paper we present a unified nomenclature for the formulation of both problems. Section 2 focuses on the single-project case (RCPSP) and Section 3 is devoted to the multi-project case (RCMPSP). In Section 4 we present the conclusions of this work.

2. RCPSP conceptual framework

The RCPSP can be conceptually formulated as follows:

A project i with a set J_i={1,...,j,..., n_i} of $|J_i|=n_i$ activities and a set L_i={1,...,l,..., k_{il}} of $|L_i|=k_{il}$ (local) renewable resources are given, where each resource l has a maximum capacity ailt=ail that is constant at each time interval t over the whole temporal horizon of the scheduling T_i. Every activity j has only one mode of execution , with a fixed duration d_{ij} in which a constant amount $r_{ijlt}=r_{ijl}$ of resource l is required for each time interval t. Ji includes the dummy activities 1 and ni that represent the start and end of the whole project, both with duration zero ($d_{i1}=d_{ini}=0$) and zero resource consumption ($r_{i1lt}= r_{inilt}=0$). All quantities d_{ij}, r_{ijl} and ail are assumed to be non-negative integers and no preemption is allowed (once an activity starts, it is not interrupted during its execution). Each activity j is linked to a set P_{ij} = {1,…,j,…,q} of q Ɛ Ji immediate predecessor activities that must be completed before the execution of j.

The typical objective function or performance measure is to find the set S_i={ST_{i1},...,ST_{ij},..., ST_{ini}} of feasible starting times for the i project activities (ST_{ij}) such that the precedence and resource constraints are met and the project completion time or makespan (MS_i) is minimized. This is equivalent to minimizing the completion date of the end project activity (ED_i end). For the temporal planning horizon T_i, which limits the space of search, an estimated upper bound is normally set for the project completion time.

With the previous definitions, the RCPSP can be formally stated as the Integer Linear Programming model (ILP) shown in Table 1.

Table 1. The RCPSP formally stated as an Integer Linear Programming model (ILP)

Objective Function:

$$\text{find } S_i = \{ST_{i1}, \dots, ST_{ij}, \dots, ST_{in_i}\}$$

such that

$$minimize\ MS_i \quad (\approx min.\ ST_{i\,end}) \ \ (\approx min.\ (\max_{j\in J_i}(ST_{ij} + d_{ij}))) \quad (1)$$

Subject to

$$ST_{ij} \geq ST_{iq} + d_{iq} \qquad \forall j,q \in J_i \ / \ q \in P_{ij} \quad (2)$$

$$ST_{ij} = ST_{i\,start} = AD_i \qquad / \ AD_i \geq 0 \quad (3)$$

$$\sum_{j\in J_{it}} r_{ijlt} \leq R_{ilt} = R_{il}(t) = R_{il} \qquad \forall\, l \in L_i\,;\ t \in \{AD_i, \dots, \overline{T}_i\} \quad (4)$$

The expression (1) represents the objective function or performance measure to optimize, which normally consists in minimizing the project makespan (i.e. the total project duration). The precedence relations between the project i activities are expressed in (2), since (3) forces the dummy start activity to begin at the Arrival Date ADi Arrival (in RCPSP generally a STi start=ADi=0 is considered). Finally, (4) limits the local resources demand imposed by the activities being processed at time t to their available capacities.

The main issue with this formulation is that in cannot be solved in a direct way, since it is not trivial to introduce the set Jit (which contains the project i's activities that are active in time t) inside the formulation of the linear problem (Demeulemeester and Herroelen 2002a).This is why other multiple alternative Mixed Integer Linear Programming (MILP) formulations for the RCPSP have been proposed in an attempt to specify the resource constraints as a function of the set Jit activities in a more solvable form: Alvarez-Valdés Olaguíbel & Tamarit Goerlich (1993); Artigues, Koné, Lopez, & Mongeau (2015); Kaplan (1988); Klein (2000); Koné, Artigues, Lopez, & Mongeau (2011, 2013); Mingozzi, Maniezzo, Ricciardelli, & Bianco (1998); Pritsker, Watters, & Wolfe (1969); Pritsker & Watters (1968).

3. RCMPSP conceptual framework

Extending the nomenclature previously used for the RCPSP in Section 2), the RCMPSP can be conceptually formulated as follows:

A set of projects I={1,...,i,..., m} of |I|=m projects and a set K_I={1,...,k,..., kI} of $|K_I|=k_I$ global renewable resources are given. Each global resource k can be shared by any of the projects and it is limited to a total availability

$R_{kt} = R_k(t) = R_k$ at each time interval t, which is constant over the whole scheduling horizon T_I.

For each project i, a set $J_i=\{1,...,j,..., ni\}$ of $|J_i|=ni$ activities and a set $L_i=\{1,...,l,..., k_{il}\}$ of $|L_i|=k_{il}$ local renewable resources are given. The resources in Li can be allocated exclusively to activities in the project i. Each local resource l Ɛ Li is constrained to a maximum capability $R_{ilt}=R_{il}(t)=R_{il}$ that is constant at each time interval t of T_I. Every activity j in project i has a duration dij; its execution requires a constant amount $r_{ijlt}=r_{ijl}(t)=r_{ijl}$ of each local resource l Ɛ Li for each time interval and a constant amount $r_{ijkt}=r_{ijk}(t)=r_{ijk}$ of each global resource k Ɛ KI for each time interval.

Within each project, the set of activities Ji includes the dummy activities 1 and ni that represent the start and the end of the whole project, both with zero duration ($d_{i1}=d_{ini}=0$)and zero resource consumption ($r_{i1lt}= r_{inilt}=r_{i1kt}= r_{inik}=0$). In addition, two dummy activities with the same properties are considered for the set of projects I: a start activity as the predecessor of all the start activities of the projects; and an end activity as the successor of all the end activities of all the projects. The first activity represents the beginning date of the schedule (ST_I start) and the second activity represents its completion date (ED_I end).

The quantities dij, rijk, rijl, ak and ail are assumed to be non-negative integers and no preemption is allowed (once an activity starts, it is not interrupted during its execution). Each activity j that belongs to the project i is linked to a set $P_{ij} = \{1,…,j,…,q\}$ of q Ɛ Ji of immediate predecessors activities that must be completed before the execution of j.

The typical objective function is to find the set $S_I=\{ST_I$ start,ST_{11},...,ST_{1j},...,ST_{1n1},…,ST_{i1},...,ST_{ij},...,ST_{ini},…,ST_{m1},...,ST_{mj},...,ST_{mnm},ST_I end} of feasible starting times for project activities (ST_{ij}) such that the precedence and resource constraints are met and the completion time or total makespan of the set of projects I (TM_{SI}) is minimized. This means minimizing the ending date of the dummy activity that represents the completion of the set of projects (ED_I end). If a solution set SI can be found, the feasible schedule for each project is directly extracted from it. For the temporal planning horizon T_I, which limits the space of search, an estimated upper bound is normally set for the completion time of the set of projects.

With the previous definitions, the RCMPSP can be formally stated as the Integer Linear Programming (ILP) model shown in Table 2.

Table 2. The RCMPSP formally stated as an Integer Linear Programming model (ILP)

Objective Function:

find $S_I = \{ST_{I\,start}, ST_{11}, \dots, ST_{1n_1}, \dots, ST_{ij}, \dots, ST_{m1}, \dots, ST_{mn_m}, ST_{I\,end}\}$

such that

$$minimize\ TMS_I \qquad (\approx min.\ ST_{I\,end})\ (\approx min.\ (\max_{j\in J_i, i\in I} (ST_{ij} + d_{ij}))) \quad (5)$$

Subject to

$$ST_{ij} \geq ST_{iq} + d_{iq} \qquad \forall i \in I;\ j,q \in J_i\ /\ q \in P_{ij} \quad (6)$$

$$ST_I = \min_{i\in I} ST_i = \min_{i\in I} ST_{i\,start} = AD_I \qquad /\ AD_I \geq 0 \quad (7)$$

$$\sum_{j\in J_{it}} r_{ijlt} \leq R_{ilt} = R_{il}(t) = R_{il} = const. \qquad \forall l \in L_i;\ i \in I;\ t \in \{AD_I, \dots, \overline{T}_I\} \quad (8)$$

$$\sum_{i\in I, j\in J_{it}} r_{ijkt} \leq R_{kt} = R_k(t) = R_k = const. \qquad \forall k \in K_I;\ t \in \{AD_I, \dots, \overline{T}_I\} \quad (9)$$

The expression (5) represents the objective function or performance measure to optimize, which normally consists in minimizing the portfolio total makespan, or equivalently, the end date of the portfolio's end dummy activity. The precedence relations between each project i activities are expressed in (6), since (7) forces the portfolio's dummy activity to begin at the Arrival Date AD_i Arrival (in RCMPSP generally a ST_I start=AD_I=0 is considered). Finally, the conditions (8) and (9) limit the local and global resources demand, respectively, imposed by the activities being processed at time t to their available capacities.

Other multiple alternative Mixed Integer Linear Programming (MILP) formulations for the RCMPSP have been proposed, that generally are extensions from RCPSP´s MILP formulations (Lova and Tormos 2001; Kyriakidis et al. 2012).

4. Conclusions

The RCPSP and the RCMPSP have become the standard problems in the literature regarding Project Scheduling. Its basic formulation serves as a starting point for a wide range of extensions. These extensions normally consist of a modification of one or several characteristics of the basic problem such as the objective function (e.g. total makespan, average project delay, resource consumption, etc.), types and characteristics of the resources (e.g. renewable or not renewable), the manner in which resources are allocated to the activities (single-mode or multi-mode scheduling), etc. In most of the cases, the same approaches and strategies used to solve the basic general problem can be readapted to solve a more complex particular case. Although several formulations have been proposed for the single and multi-project scheduling problems (RCPSP and RCMPSP), we have observed that different authors have traditionally used different nomenclatures for similar approaches, which hinders the comparison between the different extensions. For this reason, in this paper we have presented a unified nomenclature for the formulation of both problems, which we hope facilitates the overall view of the foundations of the formulation of the RCPSP and the RCMPSP as an Integer Linear Programming (ILP) model.

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